# **Phantom appearance of non-phantom matter**

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**Abstract.** Two cosmological models with non-phantom matter having the same expansion of the universe as phantom cosmologies are constructed. The first model is characterized by the evolving gravitational "constant" G and a dark energy component with a non-conserved energy-momentum tensor. The second model includes two interacting components, the dark energy component and the matter component. Closed form solutions are obtained for the constant values of model parameters and constraints on the parameters of each model from cosmological observations are outlined. For both models it is explicitly shown how the components of each model produce the expansion of the universe characteristic of phantom cosmologies, despite the absence of phantom energy. These findings stress the interpretation of phantom energy as an effective description of the more complex dynamics of non-phantom matter.

# **1 Introduction**

Complementary cosmological observations of supernovae of the type Ia (SNIa) [1], cosmic microwave background radiation (CMBR) [2], large-scale structure (LSS) [3] and other cosmic phenomena have firmly established the picture of the accelerated expansion of the contemporary universe [4]. The present accelerating phase of the expansion of the universe and its onset at a relatively low redshift  $(z \sim 1)$  represent one of the most intriguing and most studied problems in modern cosmology. The majority of theoretical explanations of this phenomenon invoke a new component of the universe named *dark energy*<sup>1</sup> with the equation of state (EOS)

$$
p_{\rm d} = w\rho_{\rm d} \,,\tag{1}
$$

where  $\rho_d$  and  $p_d$  represent dark energy density and pressure, respectively. The most studied theoretical candidates for the role of dark energy are the cosmological constant (CC)  $(w = -1)$  [6–8], and its dynamical variants such as the renormalization group running CC [9–11], quintessence  $(w \ge -1)$  [12], tachyon models [13]  $(w \ge -1)$ and the Chaplygin gas model  $(w \ge -1)$  [14].

Recent analyses of cosmological observations [15–19] allow, and even favor, a sort of dark energy with a supernegative EOS, i.e.  $w < -1$ . This unorthodox type of dark energy, first introduced in [20], was named *phantom energy*. Many models of phantom energy appeared soon [21], addressing both its fundamental implications and cosmological consequences. One of the most interesting features of phantom energy is certainly the possibility of the divergence of the scale factor of the universe in finite time. The expansion of the universe in such a model of phantom energy leads to the unbounding of all bound structures, a phenomenon also vividly referred to as "big rip" [22].

Although phantom energy represents a phenomenologically appealing possibility, the violation of the dominant energy condition (DEC), inherent in phantom energy models, leads to problems at the microscopic level. For example, it is possible to describe phantom energy in terms of the effective scalar field theory with negative kinetic terms, valid up to some cut-off scale. In such a formulation the vacuum of the theory is no longer stable, i.e. phantom energy decays. Such theories can still be cosmologically viable if the lifetime of phantom energy surpasses the age of the universe. This requirement puts stringent constraints on the parameters of the effective scalar filed theory, above all on its cut-off scale [23, 24]. There still remains a question whether some other viable microscopical formulation exists.

In such a conflict between favor from the observational side and disfavor from the theoretical side, phantom energy models face an interesting alternative: the *possibility that matter which has no phantom characteristics (e.g. satisfies DEC) produces observational effects attributed to phantom energy*. In this paper we consider two realizations of this possibility. The first realization given in Sect. 2 is a model reminiscent of *generalized phantom energy* [25], in which we consider a sort of cosmology with a timedependent gravitational "constant" G and a dark energy component with a non-conserved energy-momentum tensor. The second realization is based on the dynamics of

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<sup>1</sup> There are alternative explanations rooted in brane-world models which do not require dark energy [5].

two interacting cosmological components and is displayed in Sect. 3.

## **2 A model with an evolving G**

We consider a cosmological model with two components. The first component, which we call the matter component, has the equation of state

$$
p_{\rm m} = \gamma(a)\rho_{\rm m} \,, \tag{2}
$$

where  $\gamma(a) \geq 0$ , and  $\rho_m$  and  $p_m$  denote the energy density and pressure of the first component, respectively. We assume that the energy-momentum tensor of this component is conserved,  $T^{\mu\nu}_{m;\nu} = 0$ , which leads to a well-known relation for the scaling of  $\rho_m$  with the scale factor  $a^2$ .

$$
\rho_{\rm m} = \rho_{\rm m,0} \; \mathrm{e}^{-3 \int_{a_0}^a (1 + \gamma(a')) \frac{da'}{a'}} \,. \tag{3}
$$

The second component, which we call the dark energy component, satisfies DEC and has the equation of state

$$
p_{\rm d} = \eta(a)\rho_{\rm d}\,, \qquad \eta(a) \ge -1\,. \tag{4}
$$

Here  $\rho_d$  stands for the energy density and  $p_d$  denotes the pressure of the dark energy component. We assume that the energy-momentum tensor of the dark energy component is not conserved, i.e.  $T^{\mu\nu}_{d;\nu} \neq 0$ . Therefore, the parameter of EOS (4) does not determine the scaling of  $\rho_d$  with a. One possible way of harmonizing non-conservation of  $T_{\rm d}^{\mu\nu}$  with the general covariance of the Einstein equation is the promotion of the gravitational constant  $G$  into a time-dependent function  $G(t)$  (see [25] for details)<sup>3</sup>.  $G(t)$ satisfies a generalized conservation condition

$$
(G(t)T^{\mu\nu})_{;\nu} = 0, \tag{5}
$$

where  $T^{\mu\nu} = T^{\mu\nu}_{m} + T^{\mu\nu}_{d}$ . It is important to stress that the procedure explained above does not represent some trivial multiplication of the constant  $G$  by some function of time  $f(t)$  and multiplication of the total energy-momentum tensor  $T^{\mu\nu}$  by  $f(t)^{-1}$  since the energy-momentum tensor of the matter component is conserved. The relation (5) can be expressed as

$$
d(G\rho_d) + \rho_m dG + 3G\rho_d (1 + \eta(a)) \frac{da}{a} = 0.
$$
 (6)

This equation determines the dynamics of G in terms of energy densities and parameters of EOS of the components of the universe. At this place, it is important to notice that in the Friedmann equation

$$
\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi}{3}G(\rho_m + \rho_d),\tag{7}
$$

the evolutions of both G and the energy densities  $\rho_{\rm m}$  and  $\rho_d$  determine the kinematics of the universe, i.e. the function  $a(t)$ . The aim of this section is to investigate the possibility that the product  $G(\rho_d+\rho_m)$  has a component which grows with the scale factor (i.e. its effective parameter of EOS is smaller than  $-1$ ), while the components of the universe have a non-phantom nature (they satisfy DEC).

To this end, we introduce an assumption of the following scaling behavior:

$$
G\rho_{\rm d} = G_0 \rho_{\rm d,0} \left(\frac{a}{a_0}\right)^{-3(1+w(a))},\tag{8}
$$

where  $w(a) < -1$ . This assumption clearly introduces a source into the Friedmann equation (7) which is identical with the source originating from phantom energy with the parameter of EOS  $w(a)$  in a model with constant G.

The evolution equation for G becomes

$$
dG =
$$

$$
\frac{3G_0\rho_{\rm d,0}}{\rho_{\rm m,0} \exp\left(-3\int_{a_0}^a (1+\gamma(a'))\frac{\mathrm{d}a'}{a'}\right)} \left(\frac{a}{a_0}\right)^{-3(1+w(a))-1}
$$

$$
\times \left[\frac{a}{a_0} \ln\left(\frac{a}{a_0}\right) \mathrm{d}w(a) + (w(a)-\eta(a)) \mathrm{d}\left(\frac{a}{a_0}\right)\right]. \tag{9}
$$

Generally, it is not possible to solve this equation in closed form, so further in this section we consider a simplified model with  $\gamma(a) = \gamma = \text{const}, \eta(a) = \eta = \text{const}$  and  $w(a) = w = \text{const}$ , to gain deeper insight via an analytical solution which one can obtain in this case. The function  $G$  can now be expressed in terms of  $a$  as

$$
G = G_0 \left( 1 - \frac{\rho_{d,0}}{\rho_{m,0}} \frac{\eta - w}{\gamma - w} \left[ \left( \frac{a}{a_0} \right)^{-3(w - \gamma)} - 1 \right] \right). \tag{10}
$$

Once we have the expression for  $G$ , we can give the expression for the other source term in the Friedmann equation (the first is given by  $(8)$ ):

$$
G\rho_{\rm m} = G_0 \left(\rho_{\rm m,0} + \rho_{\rm d,0} \frac{\eta - w}{\gamma - w} \right) \left(\frac{a}{a_0}\right)^{-3(1+\gamma)}
$$

$$
-G_0 \rho_{\rm d,0} \frac{\eta - w}{\gamma - w} \left(\frac{a}{a_0}\right)^{-3(1+w)},\tag{11}
$$

while the total source term (the right-hand side of the Friedmann equation) becomes

$$
G(\rho_{\rm m} + \rho_{\rm d}) = G_0 \left( \rho_{\rm m,0} + \rho_{\rm d,0} \frac{\eta - w}{\gamma - w} \right) \left( \frac{a}{a_0} \right)^{-3(1+\gamma)} + G_0 \rho_{\rm d,0} \frac{\gamma - \eta}{\gamma - w} \left( \frac{a}{a_0} \right)^{-3(1+w)} . \tag{12}
$$

Closer inspection of (12) shows that in our cosmological model the universe evolves as if the quantity  $G$  were constant and we had one phantom component with the parameter of EOS w and one non-phantom component with

 $2 \text{ The subscript } 0 \text{ denotes the present epoch throughout this}$ paper.

 $3$  Many models consider the time-dependent  $G$ , such as the renormalization group running of  $G$  [9–11,26], the timedependence of G originating from extra dimensions [27] or the effective  $G$  in scalar–tensor theories [28].

the parameter of EOS  $\gamma$ , both components having conserved energy-momentum tensors. Therefore, our cosmological model mimics the behavior of the model with a non-phantom matter component with the present energy density  $\tilde{\rho}_{m,0} = \rho_{m,0} + \rho_{d,0} \frac{\eta - w}{\gamma - w}$  and a phantom component with the present energy density  $\tilde{\rho}_{d,0} = \rho_{d,0} \frac{\gamma - \eta}{\gamma - w}$ . The acceleration of the universe in our model is given by

$$
\frac{\ddot{a}}{a} = \n-\frac{4\pi}{3} G_0 \left[ (1+3\gamma) \left( \rho_{\text{m},0} + \rho_{\text{d},0} \frac{\eta - w}{\gamma - w} \right) \left( \frac{a}{a_0} \right)^{-3(1+\gamma)} \n+\left( 1+3\eta - (1+3\gamma) \frac{\eta - w}{\gamma - w} \right) \rho_{\text{d},0} \left( \frac{a}{a_0} \right)^{-3(1+w)} \right].
$$
\n(13)

Let us finally consider possible constraints on the parameters of the model. The assumption of constancy of all parameters of EOS is probably an oversimplification. However, for modest variations of some of the parameters with the scale factor  $a$ , one expects that the model given above represents a good approximation. It is certainly conceivable that for the more realistic parameters  $w(a)$ ,  $\eta(a)$ and  $\gamma(a)$  one can, at the level of the Friedmann equation, obtain in our model a sort of dynamics which is identical with the dynamics in the more general model with constant G and conserved energy-momentum tensors of the phantom and non-phantom components. Given many observational constraints on the past evolution of  $G$  [29], one would expect quite stringent constraints on the difference  $\eta - w$ , which should be small. This means that this model could explain the cosmological expansion with  $w$ not much more negative than −1. However, in the general case of our model, a sort of cosmology where w would differ from −1 more substantially is certainly not excluded. The expression (13) for the acceleration of the expansion of the universe provides another constraint on the parameters of the model. Namely, in order to have a transition from deceleration to acceleration at low redshift  $(z \sim 1)$ , the coefficient  $1 + 3\eta - (1 + 3\gamma)\frac{\eta - w}{\gamma - w}$  must be negative. In the case that the difference  $\eta - w$  is small, this requirement reduces to the standard one,  $\eta < -1/3$ . The requirement of  $\eta - w$  being small also favors the possibility  $\eta = -1$ , which is equivalent to the time-dependent cosmological constant. Models with the time-dependent cosmological constant and G, studied in [30], represent a specially interesting case. Models with a growing cosmological constant (and a time-dependent  $G$ ) [31] exhibit a very peculiar fate of the universe, leading to the unbounding of all gravitationally bound systems, while leaving non-gravitationally bound systems unaffected, the so-called "partial rip" scenario.

#### **3 A model with two interacting components**

In this section we consider a model with two interacting, non-phantom components. The first component, the dark energy component, is described by the equation of state

$$
p_{\rm d} = \eta(a)\rho_{\rm d}, \qquad \eta \ge -1, \tag{14}
$$

while the other component, the matter component, is determined by the following equation of state:

$$
p_{\rm m} = \gamma(a)\rho_{\rm m} \,, \qquad \gamma \ge 0 \,. \tag{15}
$$

In this model, the gravitational constant  $G$  has no spacetime variation. The interaction of the components is included in the model in the following way. We assume that the energy-momentum tensors of the two separate components are not conserved, but the total energy-momentum tensor  $T^{\mu\nu} = T^{\mu\nu}_{m} + T^{\mu\nu}_{d}$  is conserved. In this way, there exists an exchange of energy and momentum between the two components. The requirement of the conservation of the total energy-momentum tensor can be expressed as

$$
d\rho_{\rm m} + 3\rho_{\rm m}(1 + \gamma(a))\frac{da}{a} = -d\rho_{\rm d} - 3\rho_{\rm d}(1 + \eta(a))\frac{da}{a}.
$$
 (16)

What remains to be determined is the specification of the interaction (energy-momentum exchange) between the components. The aim of this model is to demonstrate that this set-up can mimic the expansion of the universe characteristic of phantom cosmologies. Therefore we assume the following evolution law for the dark energy component:

$$
\rho_{\rm d} = \rho_{\rm d,0} \left(\frac{a}{a_0}\right)^{-3(1+w(a))},\tag{17}
$$

where  $w(a) < -1$ , i.e. the dark energy component has the evolution law characteristic of phantom energy. The non-phantom dark energy component has a scaling with a characteristic of phantom energy owing to the interaction with the matter component. Equation (16) then determines the evolution law for the energy density of the matter component. For general values of the parameters of EOS it is not always possible to obtain the solutions in closed form. Therefore, in the remainder of this section we assume that these parameters are constant, i.e.  $\gamma(a) = \gamma = \text{const}, \eta(a) = \eta = \text{const}$  and  $w(a) = w = \text{const.}$  This particular choice will allow us to gain insight via closed form solutions. The energy density of the matter component then becomes

$$
\rho_{\rm m} = \left(\rho_{\rm m,0} + \rho_{\rm d,0} \frac{\eta - w}{\gamma - w}\right) \left(\frac{a}{a_0}\right)^{-3(1+\gamma)}
$$

$$
-\rho_{\rm d,0} \frac{\eta - w}{\gamma - w} \left(\frac{a}{a_0}\right)^{-3(1+w)}.
$$
(18)

The total energy density,  $\rho = \rho_m + \rho_d$ , which appears on the right-hand side of (7), then has the form

$$
\rho = \left(\rho_{m,0} + \rho_{d,0} \frac{\eta - w}{\gamma - w}\right) \left(\frac{a}{a_0}\right)^{-3(1+\gamma)}
$$

$$
+ \frac{\gamma - \eta}{\gamma - w} \rho_{d,0} \left(\frac{a}{a_0}\right)^{-3(1+w)}.
$$
(19)

The acceleration of the expansion of the universe is given by the expression

$$
\frac{\ddot{a}}{a} = \n-\frac{4\pi}{3} G \left[ (1+3\gamma) \left( \rho_{\text{m},0} + \rho_{\text{d},0} \frac{\eta - w}{\gamma - w} \right) \left( \frac{a}{a_0} \right)^{-3(1+\gamma)} \n+\left( 1 + 3\eta - (1+3\gamma) \frac{\eta - w}{\gamma - w} \right) \rho_{\text{d},0} \left( \frac{a}{a_0} \right)^{-3(1+w)} \right]. (20)
$$

As for the model displayed in Sect. 2, the right-hand side of the Friedmann equation is the same as in a model with constant G and two non-interacting components: the first being the non-phantom component with the present energy density  $\rho_{m,0} = \rho_{m,0} + \rho_{d,0} \frac{\eta - w}{\gamma - w}$  and the parameter of EOS  $\gamma$ , while the second being phantom energy with the present energy density  $\tilde{\rho}_{d,0} = \frac{\tilde{\gamma} - \eta}{\gamma - w} \rho_{d,0}$  and the parameter of EOS  $w$ . Equation (18) shows the effects of the interaction with the dark energy component on  $\rho_{\rm m}$  as an additional term growing as  $a^{-3(1+w)}$ . The requirement that the scaling law of the matter component should not differ too much from the scaling law dictated by its EOS  $(\sim a^{-3(1+\gamma)})$ , i.e. that the interaction is not too strong, leads to the condition that the difference  $\eta - w$  should be small. In the model with a more general variation of some of the parameters  $\gamma$ ,  $\eta$  or w, it is conceivable that this constraint would be milder. Again, as in Sect. 2, two non-phantom components mimic phantom cosmology. The model can successfully describe the transition from the decelerating to the accelerating regime of the expansion of the universe if the coefficient  $1+3\eta-(1+3\gamma)\frac{\eta-w}{\gamma-w}$  is negative. When  $\eta$  is close to w, the afore-mentioned requirement reduces to the condition  $\eta < -1/3$ . One especially interesting variant of the model is the case  $\eta = -1$ . In this case, the evolving cosmological constant in interaction with the matter component mimics the expansion of phantom cosmology.

One way of elaborating the model given in this section would certainly be its formulation in terms of classical fields. We can model the system of two interacting components as a system of two minimally coupled interacting scalar fields in a cosmological setting. For the Lagrangian of the interacting system we then take a general form (we consider only time-dependent scalar fields)

$$
\mathcal{L} = \frac{\dot{\phi}^2}{2} + \frac{\dot{\psi}^2}{2} - V(\phi, \psi),
$$
 (21)

where  $\phi$  and  $\psi$  denote scalar fields. Given that the total energy density is  $\rho = \frac{\dot{\phi}^2}{2} + \frac{\dot{\psi}^2}{2} + V(\phi, \psi)$  and the total pressure is  $p = \mathcal{L}$ , one obtains the following two constraints on the dynamics of the scalar fields:

$$
\dot{\phi}^2 + \dot{\psi}^2 = (1 + \eta)\rho_d + (1 + \gamma)\rho_m ,2V(\phi, \psi) = (1 - \eta)\rho_d + (1 - \gamma)\rho_m.
$$
 (22)

From (17) and (18) we have obtained  $\rho_d$  and  $\rho_m$ , respectively, as functions of the scale factor a. On the other hand, from (7) we can determine the function  $a(t)$ . This makes the right-hand sides of (22) known functions of time. All pairs of functions  $\phi$  and  $\psi$  (with a non-trivial potential  $V(\phi, \psi)$  that satisfy (22) can produce the evolution of the universe as described in the model of this section. This class of solutions certainly does not exclude more sophisticated (and realistic) field (or microscopic) models.

## **4 Conclusions**

The two models, described in Sects. 2 and 3, have been constructed to demonstrate that cosmologies *without* phantom energy can lead to an expansion of the universe usually attributed to phantom energy. The first model is characterized by an evolving gravitational "constant" G and a dark energy component with a non-conserved energy-momentum tensor. The second model is based on two interacting components. Both models yield results for the cosmological evolution of their components which are testable against the results of various cosmological observations. Calculations in this paper have been made with a specific choice of parameters (e.g. constant parameters of EOS) which ensures closed form solutions. These solutions facilitate the interpretation of the physical meaning of the obtained results, but the scope of the models described in this paper certainly does not end here. Models with variable (e.g. dependent on a) parameters  $\gamma$ ,  $\eta$  and w offer much more possibilities (especially in terms of satisfying numerous constraints from the past evolution of the universe) and merit further investigation. The possibility of mimicking phantom cosmology by a non-phantom one, certainly does not rule out an appealing and provocative idea of phantom energy. However, it puts a greater ponder on the nature of phantom energy as an effective description of the more complex dynamics of non-phantom matter.

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